Ecole Doctorale des Sciences Fondamentales

Title of the thesis: Controllability and Inverse Problems in fluid mechanics

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Summary : The framework of the thesis is the numerical analysis of control problems and inverse problems for partial differential equations. Control theory analyzes the possibility to act on a dynamical system, through distributed or boundary actuators, in order to drive the state to a given trajectory at a positive time. Inverse theory aims to recover information on a system (for instance coefficients) from partial informations obtained from measurements.

The numerical approximation of controls problems for hyperbolic type equation is now well-understood: several technics allow to get robust and convergent approximations.

For parabolic type equations such as the heat equation, recent works have shown that the situation is more delicate. The difficulty is related to the irreversibility in time of the operators, and requires the use of non-standard regularization approaches. Precisely, for the one dimensional heat, some variational type approach based on Carleman estimates, have been introduced in the last years, leading to robust approximation of controls. In particular, these method allows to obtain the strong convergence of the approximation.

The main goal of the thesis is to consider the challenging and of practical interest case of the Navier-Stokes system, that appears in fluid mechanics. The work may be decomposed as follows :

Part 1- Study some technics in control theory, including the Carleman type approach and application to the heat equation and to Stokes system.

Part 2- Adapt [2] to the case of the Stokes system

- Propose a variational formulation
- Propose a numerical approximation and perform the corresponding numerical analysis.
- Perform the implementation in a code (Matlab, C++, Free-Fem).

Part 3- Adapt [3] to the nonlinear case of the Navier-Stokes system

Propose a fixed point scheme, prove its convergence, and perform the numerical implementation.

Part 4- Compare this approach with the least-squares type approach developed in [4,6,7].

Part 5 - Apply the results in the closed context of inverse problems [9].

This thesis requires some skills in functional analysis (PDE Theory, Calculus of Variations), numerical analysis and also in computing.

More information can be found at **http://math.univ-bpclermont.fr/~munch/**

Interested people are encouraged to contact Arnaud MUNCH.

References

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[9] N. Cindea, A. Munch, Inverse problems for linear hyperbolic equations using mixed formulations, Inverse problems 2015.