Dynamical φ^4_3 on large scales

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Paths to, from and in renormalization

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Stochastic quantisation equation

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- ξ space-time white noise, i.e. centred Gaussian $\mathbb{E}\xi(t,x)\xi(t',x') = \delta(t-t')\delta(x-x')$.
- Spatial dimension $d = 2$ or $d = 3$.
- $A \in \mathbb{R}$ real parameter.

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Invariant measure, φ^4 model, formally given by

$$\mu \propto \exp\Big(-\frac{1}{4}\int \varphi^4 + 2A\varphi^2 dx\Big)\nu(d\varphi)$$

ν distribution of Gaussian free field.

Aim of this talk

$$\partial_t \varphi = \triangle \varphi - \varphi^3 - A\varphi + \xi$$

Problem:

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Local theory available:

- $d = 2$ da Prato-Debussche ’03.
- $d = 3$ Hairer ’14, Catellier-Chouk ’14.

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Main result of this talk: Global theory

- $d = 2$ existence and uniqueness on $[0, \infty) \times \mathbb{R}^2$.
- $d = 3$ existence and uniqueness on $[0, \infty) \times \mathbb{T}^3$.

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  - Arise as scaling limits (Presutti et al. 90s , Mourrat-W. '14).
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Method in a nutshell:

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Method in a nutshell:

- Only non-linear term has right sign – strong non-linear damping term.
- Difficulty: How to extract this in presence of random distributions, infinite constants, etc.
- This is a PDE talk.

Two-dimensional case: Da Prato-Debussche 2003

Stochastic step: I solution of stochastic heat equation:

$$\partial_t \mathsf{I} = \triangle \mathsf{I} + \xi.$$

Can construct $\mathsf{I}^2 \rightsquigarrow \vee$ and $\mathsf{I}^3 \rightsquigarrow \Psi$. All $\mathsf{I}, \vee, \Psi$ distributions in $\mathcal{C}^{0-}$.

Two-dimensional case: Da Prato-Debussche 2003

Stochastic step: $\<1>$ solution of stochastic heat equation:

$$\partial_t \<1> = \triangle \<1> + \xi.$$

Can construct $\<1>^2 \rightsquigarrow \<2>$ and $\<1>^3 \rightsquigarrow \<3>$. All $\<1>, \<2>, \<3>$ distributions in $\mathcal{C}^{0-}$.

Deterministic step: $u = \varphi - \<1>$.

$$\begin{aligned}\partial_t u &= \triangle u - (\<1> + u)^3 \\ &= \triangle u - \big(u^3 + 3\<1> u^2 + 3\<2> u + \<3>\big).\end{aligned}$$

Two-dimensional case: Da Prato-Debussche 2003

Stochastic step: $\dot{\mathsf{I}}$ solution of stochastic heat equation:

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Can construct $\dot{\mathsf{I}}^2 \rightsquigarrow \ddot{\mathsf{V}}$ and $\dot{\mathsf{I}}^3 \rightsquigarrow \dddot{\Psi}$. All $\dot{\mathsf{I}}, \ddot{\mathsf{V}}, \dddot{\Psi}$ distributions in $\mathcal{C}^{0-}$.

Deterministic step: $u = \varphi - \dot{\mathsf{I}}$.

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Multiplicative inequality: If $\alpha < 0 < \beta$ with $\alpha + \beta > 0$

$$\|\tau\, u\|_{\mathcal{C}^\alpha} \lesssim \|\tau\|_{\mathcal{C}^\alpha}\, \|u\|_{\mathcal{C}^\beta}.$$

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Can construct $\Psi^2 \rightsquigarrow {:}\Psi^2{:}$ and $\Psi^3 \rightsquigarrow {:}\Psi^3{:}$. All $\Psi, {:}\Psi^2{:}, {:}\Psi^3{:}$ distributions in $\mathcal{C}^{0-}$.

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Short time existence, uniqueness via Picard iteration.

Non-explosion on the torus I

Testing against u^{p-1}

$$\frac{1}{p}\left(\|u_t\|_{L^p}^p - \|u_0\|_p^p\right) + \int_0^t \left[(p-1)\left\|u_s^{p-2}\,|\nabla u_s|^2\right\|_{L^1} + \|u_s^{p+2}\|_{L^1}\right] ds$$
$$= \int_0^t \left\langle B(u_s, \tau_s), u_s^{p-1}\right\rangle ds.$$

Use the sign of $-u^3$ to get additional "good term".

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Bad terms:

$$\left\langle B, u^{p-1} \right\rangle = \left\langle -3u^2 \uparrow - 3u \vee - \Psi, u^{p-1} \right\rangle .$$

Non-explosion on the torus II

Control bad term: $\left\langle u^2 \overset{\bullet}{|}, u^{p-1} \right\rangle = \left\langle u^{p+1}, \overset{\bullet}{|} \right\rangle$.

1. Duality:

$$\left| \left\langle u^{p+1}, \overset{\bullet}{|} \right\rangle \right| \lesssim \|u^{p+1}\|_{\mathcal{B}^{\alpha}_{1,1}} \|\overset{\bullet}{|}\|_{\mathcal{B}^{-\alpha}_{\infty,\infty}}.$$

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2. Interpolation:

$$\|u^{p+1}\|_{\mathcal{B}^{\alpha}_{1,1}} \lesssim \|u^{p+1}\|_{L^1}^{1-\alpha} \|\nabla(u^{p+1})\|_{L^1}^{\alpha} + \|u^{p+1}\|_{L^1} .$$

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$\sup_{0\le t\le T} \|\text{⫯}\|_{\mathcal{B}^{-\alpha}_{\infty,\infty}}$ finite by construction. The terms $\|u^{p+1}\|_{L^1}^{1-\alpha}$ and $\|\nabla(u^{p+1})\|_{L^1}^{\alpha}$ are controlled by good terms.

Yields a priori bound on $\|u\|_{L^p}$, enough for non-explosion.

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- We expect to be able to show tightness of orbits in Krylov Bogoliubov scheme $\Rightarrow$ alternative construction of invariant measure.
- Cubic $-\varphi^{:3:}$ could be replaced by any Wick polynomial with odd degree.
- Related (but different) construction for PAM on $\mathbb{R} \times \mathbb{R}^3$ by Hairer, Labbé '15.

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$$\partial_t u = \triangle u - \left(u^3 + 3\Psi u^2 + 3\Psi^2 u + \Psi^3 \right)$$

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- Next order expansion $u = \varphi - \Psi + \mathcal{I}(\Psi^3)$ gives

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  Still cannot be solved, because of $\Psi^2 u$. Expanding further does not solve the problem.

System of equations with paraproducts

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$$v \circledequal \vee = -3\left[(v + w - \Psi) \varolessthan \mathrm{Y}\right] \circledequal \vee + \mathrm{com}_1(v, w) \circledequal \vee$$

$$= -3(v + w - \Psi)\,\mathrm{Y}^{\circledequal}_{\vee} + \mathrm{com}_2(v + w) + \mathrm{com}_1(v, w).$$

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$$= -3(v + w - \Psi)\,\Upsilon\!\!\vee + \mathrm{com}_2(v + w) + \mathrm{com}_1(v, w).$$

Comment: Very similar to Hairer's regularity structures.

Discussion of terms

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\begin{cases}
(\partial_t - \triangle)v = -3(v + w - \Psi) \varolessthan \vee, \\
(\partial_t - \triangle)w = -(v + w)^3 - 3\mathrm{com}_1(v, w) \circledcirc \vee - 3w \circledcirc \vee \\
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- $w \circeq \vee$ linear in w, but derivative or order $1+$ needed to control this.
- $a_2(v+w)^2$ nonlinear bad term. $a_2 \in \mathcal{C}^{-\frac{1}{2}-}$.

Sketch of strategy for non-explosion

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\begin{cases}
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Step 1: Gronwall-type argument bounds v in terms of w.

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Does not yet yield self-consistent bound because of $w \circledcirc \vee$ which requires $1+$ derivatives to control. We get $\int_0^t \|w\|_{L^6}^6 \lesssim \int_0^t \|w\|^2_{\mathcal{B}_2^{1+2\varepsilon}} ds + \dots$.

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Does not yet yield self-consistent bound because of $w ⊜ \vee$ which requires $1+$ derivatives to control. We get $\int_0^t \|w\|_{L^6}^6 \lesssim \int_0^t \|w\|^2_{\mathcal{B}_2^{1+2\varepsilon}} ds + \ldots$.

Step 4: Gronwall type argument for $\int_0^t \|w\|^2_{\mathcal{B}_2^{1+2\varepsilon}} ds$.

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- Theory on $\mathbb{R}^3$.

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Outlook:

- Theory on $\mathbb{R}^3$.
- Establish bounds that are uniform in $t \Rightarrow$ alternative construction for stationary φ_3^4 theory.
  Method completely different from Glimm-Jaffe '73.